

Conditional Fault Diagnosis of Bubble Sort Graphs under the PMC Model *

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Abstract: As the size of a multiprocessor system increases, processor failure is inevitable, and fault identification in such a system is crucial for reliable computing. The fault diagnosis is the process of identifying faulty processors in a multiprocessor system through testing. For the practical fault diagnosis systems, the probability that all neighboring processors of a processor are faulty simultaneously is very small, and the conditional diagnosability, which is a new metric for evaluating fault tolerance of such systems, assumes that every faulty set does not contain all neighbors of any processor in the systems. This paper shows that the conditional diagnosability of bubble sort graphs B_n under the PMC model is $4n - 11$ for $n \geq 4$, which is about four times its ordinary diagnosability under the PMC model.

Keywords: Conditional diagnosability; Fault diagnosis; Bubble sort graphs.

1 Introduction

With the rapid development of multi-processor systems, fault diagnosis of interconnection networks has become increasingly prominent. As a significant increase in the number of processors, processor failure is inevitable. In order to ensure the stable running of the systems, we must find out the faulty processors and repair or replace them. System-level diagnosis, as a powerful tool, has been widely used. The basic idea is to design an effective algorithm to find out faulty processors through a comprehensive analysis of test results which are stimulated by adjacent processors. This method does not have to use special equipment to test.

Most of the recent research efforts in system-level diagnosis have focused on enhancing the applicability of system-level diagnosis-based approaches to practical scenarios such as VLSI testing [8], diagnosis of interconnection networks employed in parallel computers [11, 12]. The classical diagnosability of a system is small owing to the fact that it ignores the unlikelihood of the corresponding processors failing at the same time. Therefore, it is attractive to develop more different measures of diagnosability based on application environment, network topology, network reliability, and statistics related to fault patterns. The self-diagnosis of system is implemented without additional cost which has a very high value in practice.

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The PMC model, proposed by Preparata *et al.* [15] for dealing with the System's self-diagnosis, assumed that each node can test its neighboring nodes, and test results are "faulty" or "fault-free". Under this model, the diagnosability of an interconnection network is the maximum number of faulty nodes in the system that can be identified. To grant more accurate measurement of diagnosability for a large-scale processing system, Lai *et al.* [11] introduce the conditional diagnosability of a system under the PMC model, which suppose the probability that all adjacent nodes of one node are faulty simultaneously is very small. That is, conditional diagnosability is the diagnosability under the condition that all adjacent nodes of any node can't be faulty simultaneously. They further showed that the conditional diagnosability of Q_n is $4(n - 2) + 1$ for $n \geq 5$. Xu *et al.* [18] established the conditional diagnosability of matching composition networks MCN under PMC model, which generalized the result on BC networks investigated by Zhu [22]. Xu *et al.* [19] studied the conditional diagnosability of Shuffle-cubes under the PMC model. Zhu *et al.* [22] showed that the conditional diagnosability of folded hypercubes FQ_n is $4n - 3$ for $n \geq 8$. Recently, Fan *et al.* [7] have derived the diagnosability of DCC linear congruential graphs under the precise and pessimistic strategies based on the PMC diagnostic model. N.W. Chang, S.Y. Hsieh [3] studied the conditional diagnosability of augmented cubes under the PMC model.

This paper establishes the conditional diagnosability of the bubble sort graph B_n under the PMC model. The remainder of this paper is organized as follows. In Section 2, we introduce some terminology and preliminaries used through this paper. Section 3 concentrates on the conditional diagnosability of B_n . Section 4 concludes the paper.

2 Terminologies and Preliminaries

For notation and terminology not defined here we follow [20]. A multi-processor system, whose topological structure is an interconnection network, can be modeled as a simple undirected graph $G(V, E)$, where a vertex $u \in V$ represents a processor and an edge $(u, v) \in E$ represents a link between vertices u and v . If at least one end of an edge is faulty, the edge is said to be faulty; otherwise, the edge is said to be fault-free. The connectivity of a graph G , denoted by $\kappa(G)$, is the minimum number of vertices whose deletion results in a disconnected graph or a trivial graph. The components of a graph G are its maximal connected subgraphs. A component is trivial if it has no edges; otherwise, it is nontrivial. The neighborhood set of the vertex set $X \subset V(G)$ is defined as $N_G(X) = \{y \in V(G) \mid \exists x \in X \text{ such that } (x, y) \in E(G)\} - X$. For convenience, $|S|$ denotes the number of elements in the set S . And we also use $|G|$ to represent the number of vertices in the graph G .

The *PMC* model requires that u and v can test each other for any edge $(u, v) \in E(G)$. When u tests v , we call u as testing node, and call v as tested node. The test output is 0 (or 1) which implies that v is faulty (or fault-free). $\sigma(u, v)$ denotes the output of u testing v . And it is assumed that the test outputs are correct if the testing node is fault-free; otherwise the outputs are unreliable.

The collection of all outputs is called the syndrome σ . For a given syndrome σ , a subset of vertices $F \subset V(G)$ is said to be consistent with σ if the syndrome σ can be produced from the situation that, for $\forall (u, v) \in E$ such that $u \in V - F$, $\sigma(u, v) = 1$ if and only if $v \in F$. It means that F is a possible set of faulty nodes. Since test output produced by a faulty node is unreliable, a given set F of faulty nodes may produce different syndromes. On the other hand, different faulty sets may produce the same syndrome. Let $\sigma(F)$ represent the set of all syndromes that could be produced by F . Two distinct sets $F_1, F_2 \subset V$ are said to be distinguishable if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$; otherwise, F_1 and F_2 are said to be indistinguishable. We say that (F_1, F_2) is a distinguishable pair if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$; otherwise, (F_1, F_2) is an indistinguishable pair. We also use $F_1 \Delta F_2 = (F_1 - F_2) \cup (F_2 - F_1)$ to denote the symmetric difference of F_1 and F_2 .

Definition 1. [11, 15] A system G is said to be t -diagnosable if, a given syndrome can be produced by a unique faulty set, provided that the number of faulty nodes present in the system does not exceed t . The largest value of t , for which a given system G is t -diagnosable, is called the diagnosability of system G , denoted as $t(G)$.

Lemma 1. [11, 15] For any two distinct sets $F_1, F_2 \subset V(G)$ of graph $G = (V, E)$, (F_1, F_2) is a distinguishable pair iff there exists a vertex $u \in V(G) - (F_1 \cup F_2)$ and there exists a vertex $v \in F_1 \Delta F_2$ such that $(u, v) \in E(G)$. \square

So, if two sets F_1 and F_2 are indistinguishable, then there is no edge between $F_1 \Delta F_2$ and $V(G) - (F_1 \cup F_2)$.

Definition 2. [11] A faulty set $F \subset V(G)$ is called a conditional fault-set, if $N(v) \not\subseteq F$ for any vertex $v \in V(G)$.

Definition 3. [11] A system G is said to be conditionally t -diagnosable, if for any two distinct conditional fault-sets $F_1, F_2 \subset V(G)$ with $|F_1| \leq t, |F_2| \leq t$, (F_1, F_2) is a distinguishable pair. The largest value of t which makes system G is conditionally t -diagnosable is called the conditional diagnosability of system G , denoted as $t_c(G)$.

Lemma 2. [11] A system G is said to be t -diagnosable under the PMC model, if and only if $\forall F_1, F_2 \subset V(G)$, $F_1 \neq F_2$ with $|F_1| \leq t, |F_2| \leq t$, (F_1, F_2) is a distinguishable pair. \square

An equivalent way of stating the lemma is the following.

Lemma 3. [11] A system G is said to be t -diagnosable under the PMC model, if and only if for an indistinguishable pair of sets $F_1, F_2 \subset V(G)$, it implies that $|F_1| > t$ or $|F_2| > t$. \square

Lemma 4. Let $G(V, E)$ be a multi-processor system, and (F_1, F_2) be an indistinguishable conditional pair with $F_1 \neq F_2$, then the following two conditions hold:

- (1) $|N(u) \cap (V - (F_1 \cup F_2))| \geq 1$ for $u \in (V - (F_1 \cup F_2))$;
- (2) $|N(v) \cap (F_1 - F_2)| \geq 1$ and $|N(v) \cap (F_2 - F_1)| \geq 1$ for $v \in F_1 \Delta F_2$. \square

Let (F_1, F_2) be an indistinguishable conditional pair, and let $S = F_1 \cap F_2$. By observation, every component of $G - S$ is nontrivial. Moreover, we have

- (1) for each component C_1 of $G - S$, if $C_1 \cap (F_1 \Delta F_2) = \emptyset$, then $\deg_{C_1}(v) \geq 1$ for $v \in V(C_1)$;
- (2) for each component C_2 of $G - S$, if $C_2 \cap (F_1 \Delta F_2) \neq \emptyset$, then $\deg_{C_2}(v) \geq 2$ for $v \in V(C_2)$.

Network reliability is one of the major factors in designing the topology of an interconnection network. The hypercubes and its variants were the first major class of interconnection networks. The n -star graph (S_n for short), which proposed by Akers *et al.*, is an attractive alternative to the hypercube [1]. The bubble-sort graphs similar to the n -star graph [6], which belongs to the class of Cayley graphs, have been attractive alternative to the hypercubes. They have some good topological properties such as highly symmetry and recursive structure. In particular, the n -dimensional bubble-sort graph B_n is vertex transitive, while it is not edge transitive [13]. The connectivity of B_n is $n - 1$ and the diameter is $n(n - 1)/2$. It was shown that finding a shortest path in B_n can be accomplished by using the familiar bubble-sort algorithm [1]. K. Kaneko, Y. Suzuki [9] proposed a polynomial time algorithm for finding disjoint paths in B_n . Y. Suzuki and K. Kaneko [17] gave an $O(n^5)$ -time algorithm that solves the node-to-set disjoint paths problem in B_n . Y. Kikuchi and T. Araki [10] have shown that the bubble sort graph B_n is edge-bipancyclic for $n \geq 5$, and $B_n - F$ is bipancyclic when $n \geq 4$ and $|F| \leq n - 3$, where F is a subset of $E(B_n)$. T. Araki and Y. Kikuchi [2] showed that the bubble-sort graph B_n is hyper-hamiltonian laceable for $n \geq 4$, and B_n is still hamiltonian laceable and strongly hamiltonian laceable if there are at most $n - 3$ faulty edges. L.M. Shih *et al.* [16] showed that the bubble-sort graph is fault tolerant maximal local connected.

Now, we introduce the bubble-sort graphs. An n -dimensional bubble-sort graph is $(n - 1)$ -regular and symmetric. It has $n!$ nodes and $\frac{(n-1)n!}{2}$ edges while its connectivity and diameter are $n - 1$ and $\frac{n(n-1)}{2}$, respectively.

The bubble-sort graph can be equivalently defined by using adjacent swapping operation with the following fashion (see Fig.1).

Definition 4. (*bubble-sort graph*) An n -bubble-sort graph B_n has $n!$ nodes. Each node has a unique address, which is a permutation of n symbols $1, 2, \dots, n$. A node that has an address $u = u_1 u_2 \dots u_n$ is adjacent to node whose address is $u^i = u_1 u_2 \dots u_{i-1} u_{i+1} u_i \dots u_n$ with $1 \leq i \leq n - 1$.

Fig.1 The bubble sort graph B_4 .

A very important property of the bubble-sort graph is its recursive structure [2]. We decompose B_n into n subgraphs $B_n^i (i = 1, 2, \dots, n)$ such that each B_n^i fixes i in the last position of the label strings which represents the vertices, and so B_n^i is isomorphic to B_{n-1} . Let $S_i = S \cap B_n^i$ for $i = 1, 2, \dots, n$. For $1 \leq i \leq n$, the i th element of the label of vertex u in B_n is represented by $u[i]$. An edge $e = xy$ is called a pair-edge if $x[n] = y[n]$ and $x[n-1] = y[n-1]$. $e = x'y'$ is called the coupled pair-edge corresponding to $e = xy$, where $x'[n] = x[n-1]$, and $y'[n] = y[n-1]$. We call two edge, xx' and yy' , the coupler or two pair-edge $e = xy$ and $e' = x'y'$.

Let S be a faulty set of $V(B_n)$. Denote $A_1 = \{B_n^i \mid B_n^i \text{ contains at least } n - 2 \text{ nodes in } S\}$, and $A_2 = \{B_n^i \mid B_n^i \text{ contains at most } n - 3 \text{ nodes in } S\}$. we also denote A_2 the subgraph of B_n induced by the union of subgraphs in A_2 .

Lemma 5. $A_2 - S$ is connected.

Proof. If $|A_2| = 0$ then there is nothing to do, and so assume $|A_2| \geq 1$. If $|A_2| = 1$, then the lemma holds, since B_n^i in A_2 is $n - 2$ -connected. Assume $|A_2| \geq 2$ below. To prove the lemma, we only need to show that B_n^i and B_n^j are connected in $A_2 - S$ for any two distinct B_n^i and B_n^j in A_2 .

Obviously, each of B_n^i and B_n^j is connected. Since vertex $u = u_1 u_2 \dots u_{n-2} i j \in B_n^j$ links to vertex $v = u_1 u_2 \dots u_{n-2} j i \in B_n^i$ with the same first $n - 2$ positions of the label strings, there are $(n - 2)!$ matching edges between B_n^i and B_n^j , that is to say these edges are non-adjacent. Because $(n - 2)! > 2(n - 3)$ for $n \geq 5$, there exists at least one fault edge between B_n^i and B_n^j . Thus B_n^i is connected to B_n^j , thus $A_2 - F$ is connected. \square

3 Conditional Diagnosability of B_n

We decompose B_n into n subgraphs $B_n^i (i = 1, 2, \dots, n)$ such that all vertices of B_n^i have the same last bit i of the label strings which represents the vertices and $B_n^i \cong B_{n-1}$.

Lemma 6. $t_c(B_n) \leq 4n - 11$.

Proof. Let $e = xy$ be one pair-edge with the coupled pair-edge $e' = x'y'$. These two pair-edges with their coupler constitute a cycle of length of 4. Obviously, $|N_{B_n}(x, y, x', y')| = 4(n - 3)$. Let $F_1 = N\{x, y, x', y'\} \cup \{x, y\}$, $F_2 = N\{x, y, x', y'\} \cup \{x', y'\}$. It is easy to check that F_1, F_2 are two indistinguishable conditional fault-sets, and $|F_1| = |F_2| = 4(n - 1 - 2) + 2 = 4n - 10$. Thus, $t_c(B_n) \leq 4n - 11$. \square

We are now ready to show the conditional diagnosability of B_n is $4n - 11$ for $n \geq 5$. Let $F_1, F_2 \subset V(B_n)$, and (F_1, F_2) be an indistinguishable conditional-pair for $n \geq 5$. We shall show our result by proving that either $|F_1| \geq 4n - 10$ or $|F_2| \geq 4n - 10$.

Lemma 7. *For any two indistinguishable conditional fault-sets F_1, F_2 in B_n with $n \geq 5$, which satisfies $F_1 \neq F_2$, we have either $|F_1| \geq 4n - 10$ or $|F_2| \geq 4n - 10$.*

Proof. Since (F_1, F_2) is an indistinguishable conditional-pair, there exists no edge between $F_1 \Delta F_2$ and $B_n - (F_1 \cup F_2)$ by Lemma 1. Define $S = F_1 \cap F_2$ and let $B_n[F_1 \Delta F_2]$ be the subgraph of B_n induced by the vertex set $F_1 \Delta F_2$. We choose a maximal component C in $B_n[F_1 \Delta F_2]$ when $B_n[F_1 \Delta F_2]$ is not connected; otherwise, let $C = B_n[F_1 \Delta F_2]$. By lemma 4, we have $|C| \geq 4$. Thus, we only need to prove $\lceil \frac{|C|}{2} \rceil + |S| \geq 4n - 10$, which implies that $|F_1| \geq 4n - 10$ or $|F_2| \geq 4n - 10$.

We decompose B_n into n subgraphs $B_n^i (i = 1, 2, \dots, n)$ such that each B_n^i fixes i in the last position of the label strings which represents the vertices. Let $S_i = S \cap B_n^i$ for $i = 1, 2, \dots, n$.

If $|S| \geq 4n - 12$, then we have $\lceil \frac{|C|}{2} \rceil + |S| \geq 4n - 10$ for $|C| \geq 4$, so the lemma holds. Now, we only consider the situation $|S| \leq 4n - 13$.

Let $A_1 = \{B_n^i \mid B_n^i \text{ contains at least } n - 2 \text{ nodes in } S\}$, and $A_2 = \{B_n^i \mid B_n^i \text{ contains at most } n - 3 \text{ nodes in } S\}$. Obviously, A_1 has at most three elements by the fact that $4(n - 2) > 4n - 12$.

If $A_1 = \phi$, by lemma 5 we have $|C| \geq (n - 3)[(n - 1)! - (n - 3)] > 2(4n - 12)$ for $n \geq 5$. Thus, we have $\lceil \frac{|C|}{2} \rceil + |S| \geq 4n - 10$. Now, we consider $A_1 \neq \phi$ as follows.

Case 1. $C \cap (A_2 - S) \neq \phi$.

Since A_1 has at most three subgraphs, we have $|C| \geq (n - 3)[(n - 1)! - (n - 3)] \geq 2(4n - 10)$ for $n \geq 5$; and we arrive at the result.

Case 2. $C \cap (A_2 - S) = \phi$.

Obviously, $C \subset A_1$, $N_{A_2}(C) \subset S \cap A_2$ by the maximality of C . Now, we divide this case into three subcases below.

Subcase 2.1. **There is exactly one subgraph, say X , in A_1 .**

Since every vertex of $C \cap X$ has exactly one neighbor outside of X , we have $N_{B_n - X}(C) \subset S \cap A_2$; and so $|N_{B_n - X}(C)| = |C|$. Obviously, $N_X(C) \subset S \cap X$. Since $|C| \geq 4$, let T be a path of length three in C . Obviously, $N_X(C) \supseteq N_X(T) - (C - T)$, and so

$$|N_X(C)| \geq |N_X(T)| - (|C - T|) \geq 4n - 12 - |C|.$$

Since $S = (S \cap X) \cup (S \cap A_2)$, we have

$$|S| = |S \cap X| + |S \cap A_2| \geq |N_X(C)| + |N_{B_n - X}(C)| \geq 4n - 12.$$

Thus, we have $\lceil \frac{|C|}{2} \rceil + |S| \geq 4n - 10$.

Subcase 2.2. **There are exactly two subgraphs, say X and Y , in A_1 .**

We assume, without loss of generality, that $|C \cap X| \geq |C \cap Y|$. For any $x \in C \cap X$, $|N_C(x)| \geq 2$ by Lemma 4 and every vertex of B_n^i has exactly one neighbor outside of this subgraph, then we have $|C \cap X| \geq |C \cap Y| \geq 2$. Due to $N_X(C \cap X) \subset S \cap X$, and $C \cap X$ has at least two vertices, $|S \cap X| \geq |N_X(C \cap X)| \geq 2(n-2) - 2$. Similarly, we have $|S \cap Y| \geq |N_Y(C \cap Y)| \geq 2(n-2) - 2$. Since every pair of $S \cap X$, $S \cap Y$, $S \cap A_2$ are disjoint, we have

$$S = (S \cap X) \cup (S \cap Y) \cup (S \cap A_2) \supseteq N_X(C \cap X) \cup N_Y(C \cap Y) \cup S \cap A_2.$$

Then we have

$$|S| \geq |N_X(C \cap X)| + |N_Y(C \cap Y)| + |S \cap A_2| \geq 2(n-2) - 2 + 2(n-2) - 2 + |S \cap A_2|.$$

Thus, we have $\lceil \frac{|C|}{2} \rceil + |S| \geq 4n - 10$.

Subcase 2.3. There are exactly three subgraphs, say X , Y , Z , in A_1 .

We assume, without loss of generality, that $|C \cap X| \geq 2$ by the fact that $|C| \geq 4$. We have $|F \cap X| \geq |N_X(C \cap X)| \geq 2(n-2) - 2$. Since every vertex of B_n^i has exactly one neighbor outside of this subgraph, and every pair of $S \cap X$, $S \cap Y$, $S \cap Z$, $S \cap A_2$ are disjoint, we have

$$S = (S \cap X) \cup (S \cap Y) \cup (S \cap Z) \cup (S \cap A_2) \supseteq N_X(C \cap X) \cup (S \cap Y) \cup (S \cap Z) \cup S \cap A_2.$$

Then we have

$$|S| \geq |N_X(C \cap X)| + |S \cap Y| + |S \cap Z| + |S \cap A_2| \geq 2(n-2) - 2 + (n-2) + (n-2) + |S \cap A_2| \geq 4n - 10.$$

Thus, we have $\lceil \frac{|C|}{2} \rceil + |S| \geq 4n - 10$. □

By Lemma 6 and 7, we have

Theorem 1. *The conditional diagnosability of bubble sort graph B_n under the PMC model is $t_c(B_n) = 4n - 11$ ($n \geq 5$).* □

Theorem 2. *The conditional diagnosability of bubble sort graph B_4 under the PMC model is $t_c(B_4) = 5$.*

Proof. Let F_1, F_2 be two distinguishable fault-sets in B_4 . Denote $S = F_1 \cap F_2$ and $C \subset F_1 \Delta F_2$ be a connected component in $B_4 - S$. By Lemma 4, we have $|C| \geq 4$.

If $|S| \geq 4$, then $\lceil \frac{|C|}{2} \rceil + |S| \geq 6$. Now we suppose that $|S| \leq 3$. In the worst case, $B_4 - S$ has two components, one of which is an isolated vertex $\{u\}$, then we have $C = B_4 - S - \{u\}$, which implies $|C| = 20$. So, $\lceil \frac{|C|}{2} \rceil + |S| > 10$. Thus $t_c(B_4) \geq 5$, while $t_c(B_4) \leq 5$, hence $t_c(B_4) = 5$. □

4 Conclusion

The issue of identifying faulty processors is important for the design of multiprocessor inter-connected systems, which are implementable with VLSI. The process of identifying all the faulty processors is the system-level diagnosis. This paper establishes the conditional fault diagnosability of the bubble sort graphs under the PMC model. The conditional diagnosability is about four times the traditional diagnosability under PMC model. This method can be also applied to other complex network structure.

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